

*To the memory of
Matej Mateev (1940-2011)*

Once more on the W -loop contribution to the Higgs decay into two photons

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Abstract

The imaginary part of the Feynman amplitude of the W -loop contribution to the Higgs decay into two gammas (viewed as a function of the square of the off shell Higgs momentum) is finite and unambiguous. It is presented as the product of an invariant amplitude \mathcal{A} times a bilinear in the components of the (on shell) photon momenta factor which takes the Ward identity into account. The unsubtracted dispersion integral of \mathcal{A} is convergent and reproduces the amplitude computed by R. Gastmans, S.L. Wu and T.T. Wu [GWW]. In particular, the decoupling theorem, criticized as an unjustified assumption in a subsequent paper [SVVZ12], is obtained as a corollary. By contrast with the currently used value (computed in [SVVZ]) our calculation provides a smaller Higgs decay rate into 2 photons than the currently observed. If accepted as a true prediction of the Standard Model, it would be a first indication for the need of a New Physics.

1 Introduction

In a pair of papers [GWW1, GWW] R. Gastmans, S.L. Wu and T.T. Wu challenged earlier calculation [EGN, SVVZ] of the W-loop contribution to the Higgs boson decay into two gammas. The authors saw the origin of the discrepancy in the use of dimensional regularization. A number of authors [DP, HTW, J, MZW, SZC, SVVZ12, W14] disputed the revised result but they all used some kind of regularization (mostly dimensional again). The controversy is interpreted in [CCNS] as a manifestation of a regularization ambiguity.

Here we offer a different calculation of this contribution which uses only (absolutely) convergent integrals with no need for regularization and we confirm the result of [GWW1, GWW]. This is achieved by first computing the discontinuity of the Feynman amplitude $\mathcal{M}_{\mu\nu}$ (continued analytically in the Higgs' momentum square to the region $p^2 > 4M^2$ where M is the mass of the W-boson) and then reconstructing the real invariant amplitude as a dispersion integral.

The imaginary (or absorptive) part of the amplitude, computed via the Cutkosky rules [C, R], is finite, as usual. We present it in the form:

$$\Im \mathcal{M}_{\mu\nu}(k_1, k_2) = \frac{-3e^2 g}{8\pi^2 M} P_{\mu\nu} \mathcal{A}(\tau), \quad (1.1)$$

$$\tau = \frac{p^2}{4M^2}, \quad p = k_1 + k_2, \quad (1.2)$$

where $P_{\mu\nu}$ is a transverse bilinear combination of the (on shell) photon momenta k_1, k_2 ,

$$P_{\mu\nu} = k_{1\nu} k_{2\mu} - (k_1 k_2) g_{\mu\nu}, \quad k_1^\mu P_{\mu\nu} = 0 = k_2^\nu P_{\mu\nu}, \quad (1.3)$$

reflecting the Ward identities, and \mathcal{A} denotes the absorptive part of the invariant amplitude. The full invariant amplitude \mathcal{F} is then given by the unsubtracted dispersion integral of \mathcal{A} , which is absolutely convergent, real for $p^2 < 4M^2$, and reproduces the result of [GWW1, GWW].

Throughout the paper we work in the unitary gauge in which the one loop calculations are drastically simplified and the time-honored dispersion theoretic procedure (that goes back to Schwinger) is particularly transparent.

2 Absorptive part of the decay amplitude

We are working with physical (outgoing) photon lines with on-shell momenta k_1, k_2 , orthogonal to the corresponding polarization vectors ζ_1, ζ_2 :

$$k_1^2 = 0 = k_2^2, \quad k_{1\mu}\zeta_1^\mu = 0 = k_{2\nu}\zeta_2^\nu. \quad (2.1)$$

This means that we can ignore terms proportional to $k_{1\mu}$ or $k_{2\nu}$ in the amplitude (cf. [GWW]). The three Feynman graphs corresponding to the 1-loop W -contribution are displayed on Fig. 1 (taken from [GWW] together with the 4-momenta on the internal lines). Clearly, the contribution \mathcal{M}_3 can be obtained from \mathcal{M}_1 by exchanging the external labels:

$$\mathcal{M}_3(k_1, \mu; k_2, \nu) = \mathcal{M}_1(k_2, \nu; k_1, \mu). \quad (2.2)$$

Here and below we are using the conventions and notation of [GWW], as well as most of their calculations, both accurate and pedagogically written.

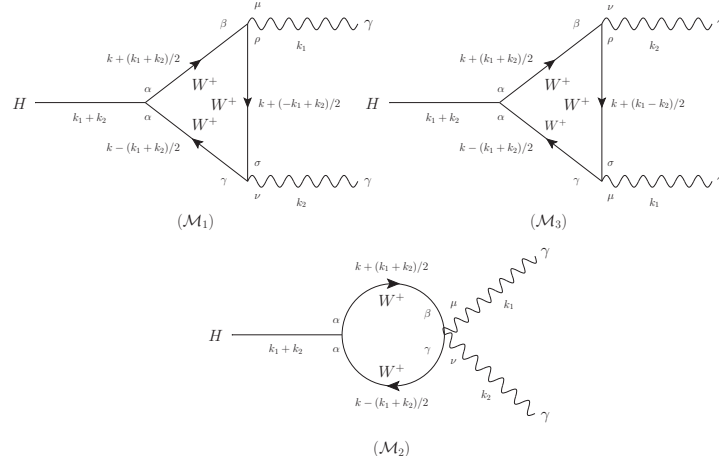


Figure 1: Feynman graphs for the W -loop contribution to the Higgs decay.

The W -propagator in the unitary gauge has the form:

$$D^{\mu\nu}(q) = -i \frac{g^{\mu\nu} - q^\mu q^\nu / M^2}{q^2 - M^2 + i\epsilon}. \quad (2.3)$$

The Cutkosky rules allow to obtain the imaginary part of the amplitude, by replacing the denominators $q^2 - M^2 + i\epsilon$ in the unitarity cut corresponding to $q = k \pm p/2$ by $-i\pi\theta(q^0)\delta(q^2 - M^2)$. If \mathcal{A}_F denotes the imaginary part of a Feynman amplitude with such a unitarity cut, then one obtains:

$$\begin{aligned}\mathcal{A}_F &= \Im \int \frac{d^4k}{(2\pi)^4} \frac{F_{\mu\nu}(k, k_1, k_2)}{[(k + \frac{p}{2})^2 - M^2 + i\epsilon][(k - \frac{p}{2})^2 - M^2 + i\epsilon]} \\ &= -\frac{1}{(4\pi)^2} \int d^4k \theta(k_0) \delta(k \cdot p) \delta\left(k^2 + \frac{p^2}{4} - M^2\right) F_{\mu\nu};\end{aligned}\quad (2.4)$$

the tensor valued function $F_{\mu\nu}$ is determined by the Feynman rules for the corresponding graphs.

In the rest frame of the (off-shell) momentum p of the decaying Higgs-boson, taking the z -axis along the space-like vector $k_1 - k_2$, the δ -functions allow to perform the integration in k^0 and $|\mathbf{k}|$ with the result:

$$\begin{aligned}p &= (\sqrt{p^2}, \mathbf{0}) \Rightarrow k_0 = 0, \quad |\mathbf{k}|^2 = \frac{p^2}{4} - M^2 = M^2(\tau - 1) \\ k &= (0, \mathbf{k}), \quad \mathbf{k} = |\mathbf{k}| (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \\ \frac{1}{\sqrt{p^2}} (k_1 - k_2) \cdot k &= -M\sqrt{\tau - 1} \cos \theta, \quad \tau > 1.\end{aligned}\quad (2.5)$$

We thus obtain:

$$\mathcal{A}_F = -\frac{\sqrt{1 - \tau^{-1}}}{(8\pi)^2} \int F_{\mu\nu}((0, \mathbf{k}), k_1, k_2) d\Omega \quad (2.6)$$

where the last integral is over the 2-dimensional sphere, $d\Omega = \sin \theta d\theta d\varphi$, $0 \leq \theta \leq \pi$, $0 \leq \varphi \leq 2\pi$ at fixed $k_0 (= 0)$ and $|\mathbf{k}|$ given in (2.5).

In the triangular graphs \mathcal{M}_1 and \mathcal{M}_3 there is one more propagator in the denominator in $F_{\mu\nu}$. It reads:

$$M^2 - (k \pm \frac{k_1 - k_2}{2})^2 = 2M^2\tau(1 \pm \sqrt{1 - \tau^{-1}} \cos \theta); \quad (2.7)$$

it does not vanish for $\tau > 1$ and hence, the integral (2.6) is well defined. Its computation (sketched in Appendix A) follows the intermediate steps of [GWW], with the advantage that all cancellations appear in an absolutely convergent (rather than superficially divergent) integral.

The resulting absorptive part \mathcal{A} of the invariant amplitude, that takes all three graphs on Fig. 1 into account, vanishes for $\tau < 1$ and is given by

$$\mathcal{A} = \frac{\pi}{2} \left(\frac{2}{\tau} - \frac{1}{\tau^2} \right) \ln \frac{1 + \sqrt{1 - \tau^{-1}}}{1 - \sqrt{1 - \tau^{-1}}} \quad \text{for } \tau \geq 1. \quad (2.8)$$

The outcome is not controversial: it agrees with both Eqs. (3.54) (3.55) of [GWW] and with the result of the earlier work [SVVZ].

3 The dispersion integral

In accord with the Ward identity for the electromagnetic interactions we define the invariant decay amplitude \mathcal{F} by:

$$\mathcal{M}_{\mu\nu}(k_1, k_2) = \frac{-3e^2 g}{8\pi^2 M} P_{\mu\nu} \mathcal{F}(\tau), \quad (3.1)$$

where $P_{\mu\nu}$ is given in (1.3).

The vanishing of the imaginary part of \mathcal{F} for $\tau < 1$ (in particular for $\tau_H = m_H^2/4M^2$), cf. (2.8), tells us that the invariant amplitude is real in the domain of interest. We identify \mathcal{F} (for $\tau < 1$) with the unsubtracted dispersion integral which is absolutely convergent:

$$\begin{aligned} \mathcal{F}(\tau) &= \frac{1}{\pi} \int_1^\infty \frac{dy}{y - \tau} \mathcal{A}(y) \\ &= \left\{ \frac{1}{\tau} + \left(\frac{2}{\tau} - \frac{1}{\tau^2} \right) \arcsin^2 \sqrt{\tau} \right\} \\ &\simeq \frac{5}{3} + \frac{22}{45} \tau + O(\tau^2) \quad \text{for } |\tau| < 1. \end{aligned} \quad (3.2)$$

(The small τ expansion in the last equation can be obtained directly from the dispersion integral using the change of variables $y = (1 - \beta^2)^{-1}$, $0 \leq \beta \leq 1$ and expanding the result around $\tau = 0$.) Eq. (3.2) agrees with the result obtained in [GWW] and differs by an additive constant, $2/3$, from the one obtained in [SVVZ] which uses dimensional regularization (DR):

$$\mathcal{F}_{DR}(\tau) = \left\{ \frac{2}{3} + \frac{1}{\tau} + \left(\frac{2}{\tau} - \frac{1}{\tau^2} \right) \arcsin^2 \sqrt{\tau} \right\}, \quad \tau \leq 1. \quad (3.3)$$

To summarize: the assumption that the invariant amplitude \mathcal{F} is given by the convergent dispersion integral *without subtraction* yields the result

of [GWW]. Adding a constant term to the dispersion integral is of course possible, as always, but, given that there is no $H\gamma\gamma$ coupling in the Standard Model Lagrangian, it does not seem to be justified by any physical requirement.

Added note. After our paper was first posted in the archive we received several comments that deserve mentioning. Roman Jackiw acquainted us with his letter to William Marciano in which he points out that the result of [GWW] can be obtained by a 4-dimensional calculation dealing with convergent integrals only, taking a surface term into account (that does not appear in dimensional regularization). Other instances of finite radiative corrections have been considered earlier in his paper [J00]. Jirí Horeisi kindly acquainted us with his and M. Stohr's paper [HS] in which a similar calculation was performed but the authors argued that the convergent dispersion integral needs a subtraction in order to fit the "Goldstone boson model" in the limit of vanishing W -mass, $M \rightarrow 0$. A similar argument is given in [SVVZ12]. It is based on the assumption that the ratio g/M , proportional to the square root of the Fermi coupling constant, stays finite for $M \rightarrow 0$. In this limit, in which $\tau \rightarrow \infty$, the absorptive part of the Higgs decay amplitude vanishes identically. We would not take the behavior in such a singular and unphysical limit as a basic requirement in the Standard Model. (Giving it often the name of "equivalence theorem", as quoted in [J], does not make it more persuasive.) Yet another plausible argument of [SVVZ], cited as a "low energy theorem", is used to fix the small τ limit of the invariant form factor \mathcal{F} . We are reluctant to take any of these (non-rigorous) arguments as a necessary complement to the Feynman rules of the Standard Model.

Concerning the worry expressed in a second version of [J] (as pointed out to us by Johannes Bluemlein), about the "worse UV singularities in the unitary gauge", we would like to reiterate that the unitary gauge is perfectly reliable when no divergences are encountered and hence no symmetry is violated. The possibility for a controversy arises because there seems to be no straightforward procedure allowing to verify the Slavnov-Taylor identities in a calculation within the unitary gauge.

We thank our correspondents, as well as John Ellis, for their interest in our work, for their criticism (even if we disagreed with some of it) and for the additional references.

4 Concluding remarks

The Higgs decay amplitude into two photons is not expected to lead to any ambiguity as there is no direct coupling between the Higgs and the photon fields in the Standard Model. The claim then that different regularizations of the W loop contribution to this process may yield different results [CCNS] is worrisome. Here we propose a dispersion theoretic calculation of the decay amplitude which deals uniquely with absolutely convergent integrals. The only assumptions involved are: 1) the extraction of a bilinear in the photon momenta factor (taking the Ward identity for the photon vertices into account) in front of the invariant amplitude \mathcal{F} , and 2) the absence of a constant term in the (convergent) dispersion integral. Both assumptions appear natural to us - being routinely made since the calculation of the photon self energy in QED.

The difference in \mathcal{F} and \mathcal{F}_{DR} , Eqs.(3.2) and (3.3), is not just of academic interest. It has a big impact on the value of the width of the decay $H^0 \rightarrow \gamma\gamma$, the best measured decay mode in the searches for the Higgs boson at LHC.

The matrix element of the decay, including diagrams with both W -bosons and t -quarks in the loops, is:

$$\mathcal{M}_{\mu\nu} = \frac{-3e^2 g}{8\pi^2 M} P_{\mu\nu} \left\{ \mathcal{F}_W(\tau) + \frac{4}{9} \mathcal{F}_t(\tau_t) \right\} \quad (4.1)$$

Here \mathcal{F}_W is the contribution from the W -boson loops. At $\tau \leq 1$, for the considered here two different approaches, it equals:

$$\begin{aligned} \mathcal{F}_W &= \mathcal{F}(\tau), & \text{Unsubtracted dispersion integral} \\ &= \mathcal{F}_{DR}(\tau) = \frac{2}{3} + \mathcal{F}(\tau), & \text{Dimensional regularization} \end{aligned} \quad (4.2)$$

where \mathcal{F} is given by (3.2). \mathcal{F}_t describes the contribution from the top-quarks diagrams. At $\tau_t = m_H^2/4m_t^2 (< 1)$, we have (see, e.g., [MZW]):

$$\mathcal{F}_t(\tau_t) = -\frac{2}{\tau_t} \left[1 + \left(1 - \frac{1}{\tau_t} \right) \arcsin^2 \sqrt{\tau_t} \right]. \quad (4.3)$$

At the measured value of the Higgs mass: $m_H \sim 125 \text{ GeV}$, we have $\tau = 0.61$ and $\tau_t = 0.13$. At this value of m_H , for the ratio of the Higgs boson widths Γ_{GWW}/Γ_{DR} of the two approaches we obtain:

$$\frac{\Gamma_{GWW}}{\Gamma_{DR}} = \frac{|\mathcal{F} + \frac{4}{9} \mathcal{F}_t|^2}{|\mathcal{F}_{DR} + \frac{4}{9} \mathcal{F}_t|^2} \simeq 0.48, \quad (4.4)$$

i.e. the predicted value in the Standard Model is reduced more than twice. The latest update of the LHC data [ATLAS, CMS, D] appears to confirm the dimensional regularization results. Accepting our calculations, one can speculate (cf. [Wu]) that this may be signaling the existence of new charged particles that contribute to the Higgs decay into two photons.

5 Appendix: Computation of $P_{\mu\nu} \mathcal{A}(\tau)$

The evaluation of the absorptive part of $\mathcal{M}_{\mu\nu}(k_1, k_2)$ is reduced (following the steps in [GWW]) to the following integrals:

$$\begin{aligned} & \Im m \int \frac{d^4 k}{(2\pi)^4} \frac{1}{\left[\left(k + \frac{p}{2} \right)^2 - M^2 + i\varepsilon \right] \left[\left(k - \frac{p}{2} \right)^2 - M^2 + i\varepsilon \right] \left[\left(k - \frac{q}{2} \right)^2 - M^2 + i\varepsilon \right]} \\ &= \frac{\beta}{8\pi p^2} I, \quad \beta = \sqrt{1 - \frac{4M^2}{p^2}}, \end{aligned} \quad (5.1)$$

$$I = \int_{-1}^{+1} \frac{dx}{1 - \beta x} = \frac{1}{\beta} \ln\left(\frac{1 + \beta}{1 - \beta}\right), \quad 0 < \beta < 1; \quad (5.2)$$

$$\begin{aligned} & \Im m \int \frac{d^4 k}{(2\pi)^4} \frac{k_\mu}{\left[\left(k + \frac{p}{2} \right)^2 - M^2 + i\varepsilon \right] \left[\left(k - \frac{p}{2} \right)^2 - M^2 + i\varepsilon \right] \left[\left(k - \frac{q}{2} \right)^2 - M^2 + i\varepsilon \right]} \\ &= \frac{\beta^2}{16\pi p^2} (k_{1\mu} - k_{2\mu}) J, \end{aligned} \quad (5.3)$$

$$J = \int_{-1}^{+1} \frac{x dx}{1 - \beta x} = \frac{1}{\beta} [I - 2], \quad (5.4)$$

$$\begin{aligned} & \Im m \int \frac{d^4 k}{(2\pi)^4} \frac{k_\mu k_\nu}{\left[\left(k + \frac{p}{2} \right)^2 - M^2 + i\varepsilon \right] \left[\left(k - \frac{p}{2} \right)^2 - M^2 + i\varepsilon \right] \left[\left(k - \frac{q}{2} \right)^2 - M^2 + i\varepsilon \right]} \\ &= \frac{\beta^3}{64\pi} \left\{ g_{\mu\nu} (K - I) + \frac{2 k_{2\mu} k_{1\nu}}{p^2} (I - 2K) \right\}, \end{aligned} \quad (5.5)$$

$$K = \int_{-1}^{+1} \frac{x^2 dx}{1 - \beta x} = \frac{1}{\beta^2} [I - 2]. \quad (5.6)$$

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